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## Quantum Computing PHYS-541, Project 8

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### *Digital error model, quantum channels, and quantum error correction*

Errors on quantum hardware can be described both with a digital error model and in a statistical model in terms of noisy quantum channels. We have seen in class that these two descriptions coincide and that the digital error model can be used to describe multiple qubit errors, thanks to the linearity of quantum mechanics. We have then developed the theory of quantum error correction based on the digital error model, which is particularly suited to this purpose. As a prototypical example of a quantum error correction code, we have studied Shor's nine-qubit error correction code. Finally, we have proven the Knill-Laflamme condition for correctable errors.

The goal of the project is to review all these concepts and to test them by implementing Shor's error correction code on a (simulated) quantum hardware on Qiskit. In particular, for this project you will:

1. Read and understand the noisy quantum channel section in Preskill's lecture notes and/or from other sources like Nielsen & Chuang's book, as well as Shor's code and the Knill-Laflamme conditions, and present them to a sufficient level of detail. For the part on Shor's code and the Knill-Laflamme conditions, you may refer to [Daniel Gottesman's PhD thesis](#) or to [this review article](#).
2. Implement Shor's nine-qubit code on the QASM simulator (don't use the built-in library for this purpose). Implement, in particular, a circuit that encodes a single qubit onto the nine-qubit code and a circuit that decodes the nine-qubit code into one qubit. Then, carry out a single qubit operation (e.g.,  $X$  or  $H$ ) transversally on the code and check that the decoded result corresponds to what is expected.
3. Simulate now an error by applying one Pauli gate to a single qubit of the nine-qubit code. Implement circuits to detect and correct  $X$  and  $Z$  errors and show that you can correct all errors. Try also using linear combinations of Pauli errors.
4. What happens if you switch on simulated noise in the QASM simulator? Is the fidelity of a single qubit, after encoding and decoding it, larger or smaller than without the error correction code? (Pay attention to the fact that an idle qubit in the QASM simulator is not subject to simulated errors, whereas a physical qubit is. In order to solve this issue, when measuring the fidelity in the absence of the QECC, an identity operation on the qubit must be executed. This will trigger simulated errors on the qubit).